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Research Article

Bayesian Inference on the Generalized Exponential Distribution Based on the Kernel Prior

Mohamed Maswadah, Seham Mohamed*

Department of Mathematics, Faculty of Science, Aswan University, Aswan, Egypt

Abstract

In this work, we introduce an objective prior based on the kernel density estimation to eliminate the subjectivity of the Bayesian estimation for information other than data. For comparing the kernel prior with the informative gamma prior, the mean squared error and the mean percentage error for the generalized exponential (GE) distribution parameters estimations are studied using both symmetric and asymmetric loss functions via Monte Carlo simulations. The simulation results indicated that the kernel prior outperforms the informative gamma prior. Finally, a numerical example is given to demonstrate the efficiency of the proposed priors.

Keywords

Informative Prior, Kernel Prior, LINEX Loss Function, Squared Error Loss Function

1. Introduction

The two-parameter Generalized Exponential (GE) distribution has Probability Density Function (PDF) and Cumulative Distribution Function (CDF), which are given respectively as:

$$f(x) = \beta\alpha e^{-\beta x}(1 - e^{-\beta x})^{\alpha-1} \tag{1}$$

$$F(x) = (1 - e^{-\beta x})^{\alpha} \tag{2}$$

α , $\beta > 0$ are shape and scale parameters respectively.

When α =1, the GE distribution reduces to the standard exponential distribution. This distribution exhibits failure rates that are both increasing and decreasing depending on the shape parameter. [1] proved that the GE distribution can be used quite effectively in analyzing many lifetime datasets, particularly in place of the two-parameter gamma and Weibull distributions. Therefore, this distribution can also be used in situations where the course of disease is such that mortality reaches a peak after some finite period, and then slowly declines as in the case of curable breast cancer. Therefore, this distribution can also be used in situations where the course of disease is such that mortality reaches a peak after some finite period and then slowly declines, as in the case of curable breast cancer. In recent years, an impressive number of studies have been written in both classical and Bayesian frameworks addressing the behavioral patterns of the parameters of the GE distribution. A very good summary of this work can be found in [2-13] and the cited there for some recent developments on the GE distribution.

The Bayesian deduction requires that the priors for the parameters be chosen adequately, since it is evident that there is no way to tell which prior is better than any other. However,

*Corresponding author: seham_mohamed@sci.aswu.edu.eg (Seham Mohamed)

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using the informative prior(s), which may be chosen over all other options, is preferable if we have sufficient knowledge about the parameter(s). This means that every conclusion drawn from the posterior distribution will vary based on the prior because the prior distribution's elicitation is based on human choice. To get around this problem, Bayesian analysis promoted the use of prior information other than data, such as knowledge of a related phenomenon or theories regarding potential values for the model's parameters or their physical meaning.

In the last decade, several authors utilized technical information about the real systems and converted it into the degree of belief about the model parameters that improved the accuracy of estimates, see [14-16].

This paper presents a suitable prior based on the theory of statistical kernel density estimators, which improves the estimation compared to the informative conjugate priors for the GE distribution parameters. The layout of this paper is as follows: In Section 2, we derive the Bayes estimates for α and β for different priors under the Squared Error Loss and LINEX Loss Functions. Monte Carlo simulation results are presented in Section 3, which provides the performance of the Bayes estimators under the Squared Error and LINEX Loss Functions in terms of the mean square and mean percentage error. An illustrative example in Section 4 is to analyze a real data set for illustrative purposes.

2. Bayesian Estimation

2.1. Informative Kernel Prior

For deriving the kernel prior, we should define the bivariate case, for the kernel density estimator for the unknown probability density function $f(x,y)$ with support on $(0,\infty)$, which is defined as follows:

$$\hat{f}(x,y)=\frac{1}{nh_1h_2}\sum_{i=1}^{n}K\left(\frac{x-x_i}{h_1},\frac{y-y_i}{h_2}\right) \tag{3}$$

h_1 and h_2 are called the bandwidths or smoothing parameters which chosen such that $h_1\to 0$, $h_2\to 0$ and $nh_1\to\infty$, $nh_2\to\infty$ as $n\to\infty$. The optimal choices for h_i which minimize the mean squared errors are $h_i=0.9S_in^{-1/7}$, $i=1,2$, where S_i the sample standard deviations of the random variables X and Y respectively, see [17]. The optimal choice for the kernel function $K(x,y)$ can be used as the bivariate standard normal distribution. Also, [18] provides a good description of the kernel estimation methods. It is important to note that the kernel function has been used in inference for the unknown parameters for some lifetime distributions, see [19-22].

As a tool for constructing the informative kernel prior, the pivotal quantities for the GE distribution parameters can be derived easily as follows:

Let $\hat{\alpha}$ and $\hat{\beta}$ be the MLEs of α and β respectively, thus we can easily write the functions:

$$Z=\alpha/\hat{\alpha} \text{ and } V=(1-e^{-\beta x_i}/1-e^{-\hat{\beta}x_i})^{\hat{\alpha}} \tag{4}$$

as pivotal quantities, which are functions of the MLEs and whose distributions are free of the unknown parameters.

The MLEs of the GE distribution parameters are obtained by differentiating the logarithm of the likelihood function of (1) and equating to zero. Thus, the two normal equations have been given below:

$$\frac{n}{\beta}-\sum_{i=1}^{n}x_i+(\alpha-1)\sum_{i=1}^{n}x_ie^{-\beta x_i}/(1-e^{-\beta x_i})=0,$$

$$n/\alpha+\sum_{i=1}^{n}ln(1-e^{-\beta x_i})=0.$$

Using an iterative technique such as the Newton-Raphson method for solving the above two equations, we can derive the MLEs from the first equation and then the MLEs from the second equation.

The methodology of statistical kernel estimators is applied for constructing the kernel prior density with the following algorithm:

1) Generate a random sample $X=(x_1,x_2,x_3,\ldots,x_n)$ from the parent distribution $f(x;\alpha,\beta)$ with a given specified values for the parameters α and β.
2) Bootstrapping with replacement n samples $X_1^*,X_2^*,X_3^*,\ldots,X_n^*$, with size n each, where $X_i^*=(x_1^*,x_2^*,x_3^*,\ldots,x_n^*)$ for $i=1,2,\ldots.,n$ from the given random sample in step 1.
3) For each sample in step 2, we can calculate the MLEs for the parameters α and β, from which and (4) we can derive the pivotal quantities $Z=(z_1,z_2,\ldots,z_n)$, and $V=(v_1,v_2,\ldots,v_n)$ whose distributions are free of the unknown parameters α and β.
4) Thus, based on the random variables Z and V we can use the kernel estimator (3) for deriving the kernel density estimator for the density function of the pivotal quantities Z and V, which is a function of the unknown parameters α and β and their MLEs and is defined as $g(z,v)$ and named as the informative kernel prior, where it contains all the information about the unknown parameters supplied by the sample.

2.2. Informative Gamma Prior

We propose the use of piecewise independent priors for both parameters, namely each of the unknown parameters α and β has gamma distribution as given by:

$$g_1(\alpha)=\frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha},\ a,b\geq 0, \text{ and } g_2(\beta)=\frac{d^c}{\Gamma(c)}\beta^{c-1}e^{-d\beta}\ c,d\geq 0.$$

Thus, the joint prior distribution is given as follows:

$$g(\alpha,\beta) \propto \alpha^{a-1}\beta^{c-1}e^{-b\alpha-d\beta} \quad (5)$$

For $a=b=c=d=0$, we get the non-informative prior.

Based on Bayes' theorem the posterior density function can be defined as

$$f(\theta|X) = \frac{g(\theta)l(\theta;X)}{\int_\theta g(\theta)l(\theta;X)d\theta}. \quad (6)$$

Here $g(\theta)$ is the prior density function and $l(\theta;X)$ is the likelihood function.

The Bayes estimators for the parameters α and β will be derived using the informative gamma prior and the non-parametric kernel prior, based on two different loss functions.

Firstly, the Squared Error Loss Function (SELF), $\mathrm{L}(\mathrm{g}(\theta),\hat{\mathrm{g}}(\theta)) = \left(g(\theta)-\hat{g}(\theta)\right)^2$. For this loss function the Bayes estimator that minimizes the risk function is given by $\hat{\mathrm{g}}(\theta) = \mathrm{E}_\theta(\mathrm{g}(\theta)|\mathrm{x})$.

Secondly, in practical applications, the underestimation of a parameter value very often implies different results from the overestimation, both in quality and quantity. Thus, these differences can be explained by a linear function with separate coefficients for positive and negative errors. This function, which is also known as the LINEX Loss Function, is defined as an asymmetric loss function in the following form:

$$L(\theta,\theta^*) = exp[\delta(\theta-\theta^*)] - \delta(\theta-\theta^*) - 1,\ \delta \neq 0.$$

The direction and degree of symmetry are represented by the sign and magnitude of the shape parameter δ, respectively. Positive values indicate that overestimation is more serious than underestimation and vice versa for negative values. The value that minimizes the risk function, or the unique Bayes estimator θ_L^* of θ under the LINEX Loss Function (LLF), is provided as

$$\theta_L^* = -\frac{1}{\delta}ln[E_\theta(e^{-\delta\theta})],$$

provided the expectation $E_\theta(e^{-\delta\theta})$ exists and is finite.

Several authors have used this function, [14, 23]. However, Bayesian estimation under the LINEX Loss Function is not frequently discussed, perhaps, because the estimator involves integral expressions, which are not analytically solvable, and one must use numerical techniques.

For the informative prior the joint posterior density is given by

$$f(\alpha,\beta|X) = K\alpha^{n+a-1}\beta^{n+c-1}exp[-\alpha(b-\textstyle\sum_{i=1}^n ln(1-e^{-\beta x_i}))]$$

$$\times exp[-\beta(d+\textstyle\sum_{i=1}^n x_i) - \sum_{i=1}^n ln(1-e^{-\beta x_i})].$$

The marginal posterior densities for the parameters can be evaluated as

$$g(\beta|X) = K\Gamma(n+a)\beta^{n+c-1}$$

$$\times [b-\textstyle\sum_{i=1}^n ln(1-e^{-\beta x_i})]^{-(n+a)}$$

$$\times exp[-\beta(d+\textstyle\sum_{i=1}^n x_i) - \sum_{i=1}^n ln(1-e^{-\beta x_i})].$$

$$f(\alpha|X) = K\int_0^\infty \alpha^{n+a-1}\beta^{n+c-1}$$

$$\times exp[-\alpha(b-\textstyle\sum_{i=1}^n ln(1-e^{-\beta x_i}))]$$

$$\times exp[-\beta(d+\textstyle\sum_{i=1}^n x_i) - \sum_{i=1}^n ln(1-e^{-\beta x_i})]d\beta\ .$$

K is the normalizing constant and can be evaluated as

$$K^{-1} = \Gamma(n+a)\int_0^\infty \beta^{n+c-1}$$

$$\times [b-\textstyle\sum_{i=1}^n ln(1-e^{-\beta x_i})]^{-(n+a)}$$

$$\times exp[-\beta(d+\textstyle\sum_{i=1}^n x_i) - \sum_{i=1}^n ln(1-e^{-\beta x_i})].$$

For the kernel prior the joint posterior density function is given by

$$f(\alpha,\beta|X) = Kg(\alpha,\beta)\alpha^n\beta^n$$

$$\times exp[-\beta(d+\textstyle\sum_{i=1}^n x_i) - \sum_{i=1}^n ln(1-e^{-\beta x_i})].$$

3. Simulation Study

The purpose of the simulation study is to assess the performance of Bayes estimates for two different loss functions that are obtained from informative gamma and informative kernel priors using the following two criteria, Mean Squared Error (MSE) and Mean Percentage Error (MPE), as given by:

$$\mathrm{MSE}(\theta^*) = \frac{1}{\mathrm{L}}\textstyle\sum_{\mathrm{i}=1}^{\mathrm{L}}(\theta^*{}_1-\theta)^2,\ \mathrm{MPE}(\theta^*) = \frac{\sum_{i=1}^M \frac{|\theta-\theta^*|}{\theta}}{M},$$

θ^* is the estimate of θ and L is the number of replications.

To assess the performance of these priors, the MSE and MPE for each prior were calculated using 1000 replications for each sample size under both the SELF and LLF. In the simulation study we generated several data sets for different combinations of the true paramter values of $\alpha = 0.5,1$ and 2 and β was set equal to 2 and 3 for sample sizes, $n = 20,\ 40$ and 80, to represent small, moderate and large sizes. It is assumed that the hyperparameters for the informative prior are $a=c=5{,}4$ and $b=d=2{,}3$. For the LINEX loss function, the shape parameter δ was set to ±1 and ±2. It may be mentioned here that, because of space restrictions, all results are not shown in the tables.

From the simulation results in Tables 2, 3 and 4, some of the points are quite clear based on the kernel and informative priors and the others have been summarized in the following main points:

1) In general, the estimated MSE and MPE values based on the kernel prior are smaller than the ones based on the informative prior under both loss functions.
2) As the sample sizes increase, the estimated MSE and MPE values for the estimates decrease based on both priors under the two loss functions.
3) In most cases, the estimated MSE and MPE values for the parameter α using the LINEX Loss Function are greater than the ones based on the Squared Error Loss Function and get smaller when the shape parameter of the LINEX Loss Function takes positive values based on the kernel prior and vice versa based on the informative prior.
4) It may be noted that the results under the LINEX Loss Function with positive values of δ are smaller than the ones for negative values of δ except for the case when $\beta = 3$, based on both priors. Moreover, these results are smaller than the ones under the Squared Error Loss Function for small values of α. Thus, there are different coefficients for positive and negative errors, which in fact proves that over and under estimation of the parameter δ has different effects on the unknown parameters.
5) The estimated MSE and MPE values for parameter α increase as the value of α increases and decrease as the value of β increases.
6) The estimated MSE and MPE values for the parameters decrease as the hyperparameters of the informative prior decrease, keeping the prior mean close to one and the variation in prior variance close to half.
7) In general, for parameter α the estimated MSE values are less than the MPE values under the Squared Error Loss Function and the LINEX Loss Function, and that is true for most cases for the parameter β.

From the results it appears that the proposed kernel prior competes and outperforms the informative prior under the Squared Error Loss Function and LINEX Loss Function.

4. An Illustrative Example

The data [23] presented here is from tests on endurance of deep groove ball bearings. The data presented are the number of million revolutions before failure for each of the 23 ball bearings in the life test and they are:

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.48, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

Figure (1a) indicates these data are good fit for the generalized exponential distribution. From the results in Table 1, the Bayesian estimates for α are close to 5, which indicates that the above dataset is moderately bell shaped, which means slightly decreasing the number of revolutions of the ball bearings before failure, see Figure (1 b). Also, the Bayesian estimates for β are almost close to zero, which ensures this dataset is almost symmetric even with increasing time. Thus, this dataset ensures the strength of the ball bearings.

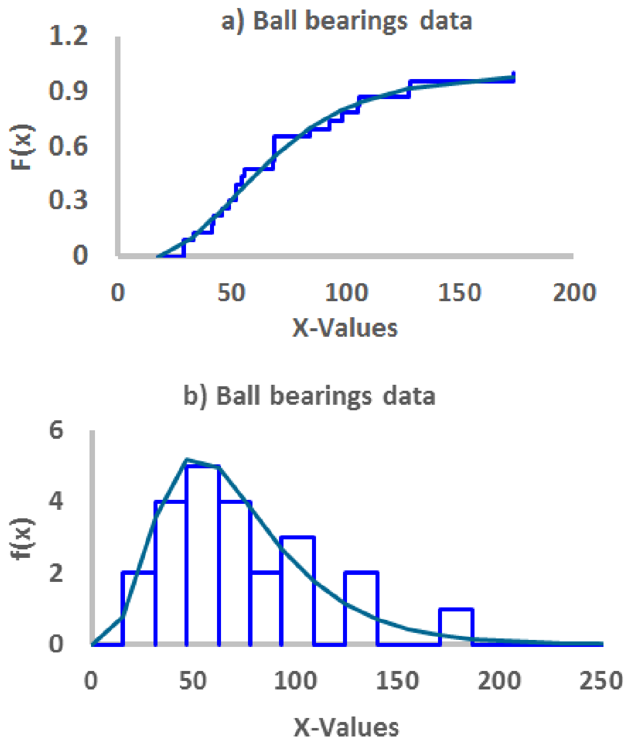

Figure 1. *a) The Empirical CDF and the fitted CDF for the data. b) The Histogram and the fitted PDF for the data.*

Figures 2 and 3 indicated that the posterior densities for α and β based on the kernel prior dominate the corresponding densities based on the informative prior, which ensuring the simulation results and the results in Table 1.

Table 1. *The Bayes estimators ($\theta_{\circ}^{*}$) for the parameters α and β for different Priors and the Mean Square Errors (MSE), Mean Percentage Errors (MPE) under Squared Error Loss (SELF) and LINEX Loss functions (LLF) with $\delta = \pm 2$, (The upper row for $\delta = 2$, the lower row for $\delta = -2$).*

	Kernel Prior						Informative Prior					
Par.	**SELF**			**LLF**			**SELF**			**LLF**		
	θ_S^*	**MSE**	**MPE**	θ_L^*	**MSE**	**MPE**	θ_S^*	**MSE**	**MPE**	θ_L^*	**MSE**	**MPE**
α	5.1451	0.0181	0.0255	5.0429	0.0559	0.0448	5.2215	0.00334	0.0109	5.0092	0.0730	0.05119
				5.0871	0.0371	0.0364				5.3834	0.0108	0.0197
β	0.0501	3.17E-04	0.5514	0.0501	3.17E-04	0.5514	0.0219	1.09E-4	0.3231	0.0218	1.09E-4	0.3241
				0.0501	3.17E-04	0.5514				0.0219	1.08E-4	0.3221

Table 2. *The Mean Square Errors (MSE) and the Mean Percentage Errors (MPE) for the parameter α for different Priors, for $\alpha = 0.5,1,2$ and $\beta = 2,3$ under the Squared Error Loss (SELF) and LINEX Loss Functions (LLF) with $\delta = 2$.*

			Kernel Prior				Informative Prior			
N	α	β	**SELF**		**LLF**		**SELF**		**LLF**	
			MSE	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**
20	0.5	2	0.0147	0.1889	0.0119	0.1724	0.0399	0.2942	0.0317	0.2616
		3	0.0097	0.1596	0.0092	0.1564	0.0240	0.2254	0.0192	0.2035
	1	2	0.0403	0.1624	0.0299	0.1419	0.0994	0.2340	0.0602	0.1850
		3	0.0332	0.1496	0.0382	0.1629	0.0572	0.1804	0.0399	0.1576
	2	2	0.1047	0.1330	0.2145	0.2160	0.1253	0.1445	0.1581	0.1692
		3	0.2136	0.2113	0.3493	0.2849	0.1557	0.1662	0.2410	0.2195
40	0.5	2	0.0085	0.1408	0.0076	0.1351	0.0147	0.1795	0.0128	0.1674
		3	0.0063	0.1285	0.0062	0.1288	0.0099	0.1480	0.0088	0.1402
	1	2	0.0323	0.1395	0.0262	0.1289	0.0507	0.1672	0.0376	0.1465
		3	0.0249	0.1294	0.0262	0.1288	0.0348	0.1412	0.0286	0.1314
	2	2	0.0749	0.1115	0.1128	0.1422	0.1136	0.1353	0.1155	0.1417
		3	0.1222	0.1496	0.1842	0.1946	0.1197	0.1431	0.1526	0.1687
80	0.5	2	0.0042	0.1018	0.0040	0.0998	0.0055	0.1142	0.0051	0.1099
		3	0.0036	0.0955	0.0036	0.0961	0.0042	0.1017	0.0039	0.0992
	1	2	0.0191	0.1091	0.0172	0.1049	0.0225	0.1167	0.0190	0.1086
		3	0.0158	0.1017	0.0163	0.1042	0.0179	0.1059	0.0162	0.1021
	2	2	0.0615	0.1014	0.0718	0.1103	0.0745	0.1106	0.0721	0.1101
		3	0.0759	0.1140	0.0997	0.1347	0.0746	0.1117	0.0849	0.1208

Table 3. *The Mean Square Errors (MSE) and the Mean Percentage Errors (MPE) for the parameter β for different priors, for $\alpha = 0.5,1,2$ and $\beta = 2,3$ under the Squared Error Loss (SELF) and LINEX Loss Functions (LLF) with $\delta = 2$.*

			Kernel Prior				**Informative Prior**			
N	***α***	***β***	**SELF**		**LLF**		**SELF**		**LLF**	
			MSE	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**
20	0.5	2	0.1058	0.1307	0.2089	0.1975	0.2378	0.1877	0.1291	0.1465
		3	0.2622	0.1385	0.4410	0.1797	0.3327	0.1604	0.7407	0.2669
	1	2	0.0878	0.1202	0.1419	0.1579	0.1784	0.1625	0.1155	0.1386
		3	0.2205	0.1294	0.3111	0.1506	0.2794	0.1461	0.5410	0.2208
	2	2	0.0910	0.1233	0.1543	0.1734	0.0954	0.1257	0.1137	0.1415
		3	0.2002	0.1246	0.2443	0.1345	0.3129	0.1597	0.5664	0.2312
40	0.5	2	0.0928	0.1247	0.2213	0.1883	0.1798	0.1657	0.1216	0.1428
		3	0.1837	0.1195	0.2467	0.1342	0.2769	0.1453	0.4592	0.1970
	1	2	0.0768	0.1125	0.1739	0.1637	0.1257	0.1402	0.0971	0.1274
		3	0.1655	0.1142	0.1834	0.1175	0.2192	0.1289	0.3195	0.1600
	2	2	0.0554	0.0963	0.1245	0.1384	0.0718	0.1096	0.0785	0.1161
		3	0.1555	0.1104	0.1527	0.1082	0.1981	0.1234	0.3045	0.1592
80	0.5	2	0.0799	0.1130	0.1306	0.1411	0.1122	0.1327	0.0875	0.1214
		3	0.1525	0.1084	0.1713	0.1129	0.1921	0.1210	0.2504	0.1401
	1	2	0.0597	0.0971	0.0906	0.1171	0.0758	0.1104	0.0644	0.1041
		3	0.1317	0.1002	0.1317	0.0993	0.1425	0.1040	0.1707	0.1144
	2	2	0.0393	0.0803	0.0664	0.1007	0.0466	0.0887	0.0477	0.0901
		3	0.1169	0.0932	0.1097	0.0904	0.1132	0.0926	0.1474	0.1068

Table 4. *The Mean Square Errors (MSE) and the Mean Percentage Errors (MPE) for the parameters α and β for different priors, for $\alpha = 0.5,1,2$ and $\beta = 2,3$ under the LINEX Loss Functions (LLF) with $\delta = -2$.*

			Kernel Prior				**Informative Prior**			
N	***α***	***β***	***α***		***β***		***α***		***β***	
			MSE	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**	**MSE**	**MPE**
20	0.5	2	0.0187	0.2100	0.1367	0.1479	0.0512	0.3334	0.7977	0.3477
		3	0.0109	0.1667	0.3049	0.1663	0.0306	0.2527	0.5771	0.1951
	1	2	0.06899	0.2076	0.1086	0.1297	0.1797	0.3123	0.4297	0.2515
		3	0.0362	0.1544	0.2061	0.1289	0.0979	0.2274	0.4169	0.1659
	2	2	0.0581	0.0976	0.0523	0.0914	0.3087	0.2099	0.1339	0.1419
		3	0.1053	0.1315	0.2291	0.1419	0.1862	0.1693	0.2158	0.1258
40	0.5	2	0.0046	0.1483	0.1367	0.1479	0.0171	0.1931	0.3777	0.2356
		3	0.0066	0.1295	0.1771	0.1135	0.0114	0.1573	0.4197	0.1685

N	α	β	Kernel Prior				Informative Prior			
			α		β		α		β	
			MSE	MPE	MSE	MPE	MSE	MPE	MSE	MPE
	1	2	0.0439	0.1597	0.1086	0.1297	0.0719	0.1969	0.2093	0.1769
		3	0.0262	0.1299	0.1217	0.0929	0.0465	0.1598	0.2876	0.1412
	2	2	0.0755	0.1111	0.0523	0.0914	0.1990	0.1673	0.0846	0.1162
		3	0.0809	0.1164	0.1222	0.0961	0.1433	0.1475	0.1619	0.1097
	0.5	2	0.0045	0.1043	0.1026	0.1249	0.0059	0.1190	0.1745	0.1609
		3	0.0036	0.0953	0.1268	0.0959	0.0045	0.1048	0.2561	0.1335
80	1	2	0.0222	0.1159	0.0723	0.1045	0.0274	0.1274	0.1019	0.1252
		3	0.01596	0.1015	0.0945	0.0834	0.0207	0.1125	0.1715	0.1107
	2	2	0.0673	0.1044	0.0387	0.0384	0.1021	0.1256	0.0516	0.0917
		3	0.0623	0.1017	0.0822	0.0785	0.0835	0.1159	0.1049	0.0887

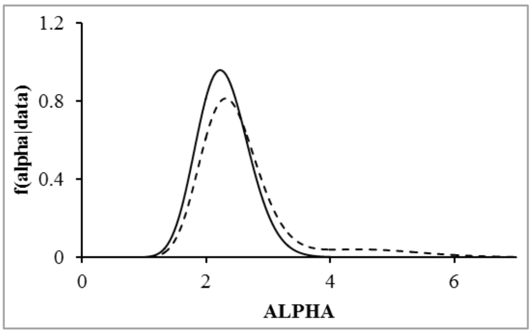

Figure 2. *The posterior densities of alpha, the kernel (solid line) and the gamma (dashed line).*

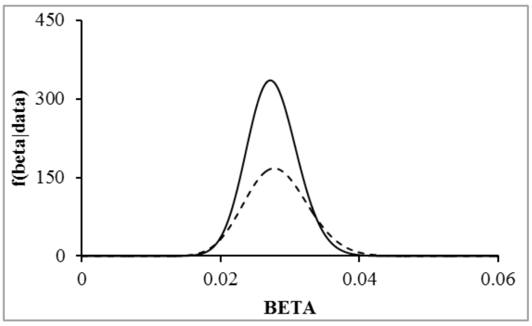

Figure 3. *The posterior densities of beta, the kernel (solid line) and gamma (dashed line).*

5. Conclusions

In statistical inference Bayes method based on the informative prior is more efficient than the other estimation methods. In this work, we used the informative kernel prior, which is more efficient and strongly unbiased than the informative gamma prior for different loss functions. The efficiency of the results based on the Kernel prior is statistically significant for researchers in the social sciences and psychology.

Abbreviations

GE: Generalized Exponential
PDF: Probability Density Function
CDF: Cumulative Distribution Function
SELF: Squared Error Loss Function
SLF: LINEX Loss Function
MSE: Mean Squared Error
MPE Mean Percentage Error

Conflicts of Interest

The authors declare no conflicts of interest.

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